

Homework 3

Physics 313

Fall 2026

Due 12:30 PM, Tuesday, 22 September 2026 as an upload to ELMS
All problems are from Griffiths *Introduction to Electrodynamics* 5th Edition

- 1) **Griffiths Problem 2.19** Hint: Use superposition. Note that the electric field inside one sphere can be written as $\vec{E} = \frac{\rho}{3\epsilon_0} \vec{r}$
- 2) **Griffiths Problem 2.22** Hint: Start with the electric field deduced from Gauss's Law
- 3) **Griffiths Problem 2.26** [Potential from simple charge distributions]
- 4) **Griffiths Problem 2.29** Hint: Remember that $(x^2)^{1/2} = |x|$
- 5) **Griffiths Problem 2.47** [Finding charge density from electric field]
- 6) **Griffiths Problem 2.49** [Potential difference of a charged bowl]

Extra Credit: Green's Theorem

Starting with the divergence theorem for a vector field $\mathbf{v}$, write $\mathbf{v}$ as $\mathbf{v} = \phi \nabla \psi$, where ϕ and ψ are scalar functions of position, and derive Green's first identity:

$$\int_V (\phi \nabla^2 \psi + \vec{\nabla} \phi \cdot \vec{\nabla} \psi) d\tau = \oint_S \phi \frac{\partial \psi}{\partial n} da$$

where we have written $\vec{\nabla} \psi \cdot \hat{n} = \partial \psi / \partial n$ to define the normal derivative at the surface, since $d\vec{a} = \hat{n} |d\vec{a}|$.

Starting from this, derive Green's second identity:

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) d\tau = \oint_S \left[\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right] da$$

Now take $\psi = 1/|\vec{r} - \vec{r}'|$ and $\phi = V$, the scalar electrostatic potential, where $\vec{r}$ is the field point and $\vec{r}'$ becomes the integration variable in the above identities. Put this into Green's second identity to develop an integral equation version of Poisson's differential equation:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' + \frac{1}{4\pi} \oint_S \left[\frac{1}{|\vec{r} - \vec{r}'|} \frac{\partial V}{\partial n'} - V \frac{\partial}{\partial n'} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \right] da'$$

The first term on the right hand side (rhs) is just that in Griffiths Eq. (2.29). However, the surface integral is something new. Provide a physical interpretation of the other two terms in the surface integral on the rhs.